Binomial Theorem

Question1

In the binomial expansion of $(p-q)^{14}$, if the sum of 7th term and 8 th term is zero, then $\frac{p+q}{p-q}=$

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Options:

A.

14

B.

15

C.

16

D.

13

Answer: B

Solution:

We are given the expression $(p-q)^{14}$. We need to find the value of $\frac{p+q}{p-q}$, given that the sum of the 7th and 8th terms in its expansion is zero.

Finding the 7th and 8th Terms

In the binomial expansion of $(p-q)^{14}$, the formula for the $(r+1)$th term is: $T_{r+1}={}^{14}C_r p^{14-r}(-q)^r$.

The 7th term, T_7, is ${}^{14}C_6 p^8(-q)^6={}^{14}C_6 p^8 q^6$.

The 8th term, T_8, is ${}^{14}C_7 p^7(-q)^7=-{}^{14}C_7 p^7 q^7$.

Setting Up the Equation

We are told that $T_7+T_8=0$.

This means: ${}^{14}C_6 p^8 q^6 - {}^{14}C_7 p^7 q^7 = 0$.

We can factor out $p^7 q^6$ from both terms: $p^7 q^6 \left({}^{14}C_6 p - {}^{14}C_7 q\right) = 0$.

Because p and q are not zero, we must have ${}^{14}C_6 p = {}^{14}C_7 q$.

Solving for $\frac{p}{q}$

Divide both sides by q and ${}^{14}C_6$ to get $\frac{p}{q} = \frac{{}^{14}C_7}{{}^{14}C_6}$.

Now, ${}^{14}C_7 = \frac{14!}{7! \cdot 7!}$ and ${}^{14}C_6 = \frac{14!}{6! \cdot 8!}$.

Write $\frac{{}^{14}C_7}{{}^{14}C_6}$ as $\frac{8}{7}$ (after cancelling out common terms, which are shown in the expansion above).

Express p and q

If $\frac{p}{q} = \frac{8}{7}$, let $p = 8k$ and $q = 7k$ for some k.

Find $\frac{p+q}{p-q}$

Then $p + q = 8k + 7k = 15k$ and $p - q = 8k - 7k = k$.

So, $\frac{p+q}{p-q} = \frac{15k}{k} = 15$.

Question2

The numerically greatest term in the expansion of $(x + 3y)^{13}$, when $x = \frac{1}{2}$ and $y = \frac{1}{3}$ is

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Options:

A.

${}^{13}C_9 \left(\frac{1}{3}\right)^4$

B.

${}^{13}C_4 \left(\frac{1}{2}\right)^9$

C.

${}^{13}C_9 \left(\frac{1}{2}\right)^4$

D.

${}^{13}C_{10} \frac{1}{2^4}$

Answer: C

Solution:

General term of $(x+3y)^{13}$ is

$T_{r+1} = {}^{13}C_r(x)^{13-r}(3y)^r$
at $x = 1/2, y = 1/3$

So, $3y = 1$

then, $T_{r+1} = {}^{13}C_r\left(\frac{1}{2}\right)^{13-r}(1)^r = {}^{13}C_r\left(\frac{1}{2}\right)^{13-r}$

For maximum term

$$\left|\frac{T_{r+1}}{T_r}\right| = \frac{{}^{13}C_r}{{}^{13}C_{r-1}}\left(\frac{1}{2}\right)^{13-r-(13-(r-1))}$$
$$= \frac{{}^{13}C_r}{{}^{13}C_{r-1}}\left(\frac{1}{2}\right)^{-1} = \frac{14-r}{r} \times 2$$
put $\left|\frac{T_{r+1}}{T_r}\right| = 1$

$\Rightarrow \quad 28 - 2r = r \Rightarrow r = 28/3 \approx 9.33$

So, maximum occurs either $r = 9$ or $r = 10$

for $r = 9$

$T_{10} = {}^{13}C_9\left(\frac{1}{2}\right)^4 = 44.6875$

and for $r = 10$

$T_{11} = {}^{13}C_{10}\left(\frac{1}{2}\right)^3 = 35.75$

Hence, the greatest term is T_{10}

$T_{10} = {}^{13}C_9\left(\frac{1}{2}\right)^4$

Question3

The coefficient of x^{10} in the expansion of $\left(x + \frac{2}{x} - 5\right)^{12}$ is

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Options:

A.

1674

B.

2132

C.

1892

D.

862

Answer: A

Solution:

Coefficient of x^{10} in the expansion of $\left(x+\frac{2}{x}-5\right)^{12}$

Using multinomial theorem

$=\frac{12!}{r_1!r_2!r_3!}x^n\left(\frac{2}{x}\right)^{r_2}(-5)^{r_3}$

for coefficient of x^{10}

$r_1+r_2+r_3=12 \Rightarrow r_1-r_2=10$

r_1	r_2	r_3
10	0	2
11	1	0

Coefficient

$$=\frac{12!}{10!0!2!}2^0\cdot(-5)^2+\frac{12!}{11!1!!}2^!.(-5)^0$$

$$=\frac{12\times 11}{2}\times 25+24$$

$$=1650+24=1674$$

Question4

Let $S_1=\sum_{j=1}^{10} j(j-1)\cdot{}^{10}C_j, S_2=\sum_{j=1}^{10} j\cdot{}^{10}C_j$ **and**

$S_3=\sum_{j=1}^{10} j^2\cdot{}^{10}C_j$

Assertion (A) $S_3=55\times 2^9$

Reason (R) $S_1=90\times 2^8$ **and** $S_2=10\times 2^8$

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Options:

A.

Both (A) and (R) are true and R is the correct explanation of (A)

B.

Both (A) and (R) are true but (R) is not the correct explanation of (A)

C.

(A) is true, but (R) is false

D.

(A) is false, but (R) is true

Answer: C

Solution:

Given,

$$S_1 = \sum_{j=1}^{10} j(j-1) \cdot {}^{10}C_j$$
$$= \sum_{j=1}^{10} j(j-1)\frac{10}{j} \cdot \frac{9}{j-1} \cdot {}^{8}C_{j-2}$$
$$= \sum_{j=2}^{10} 90 \cdot {}^{8}C_{j-2} = 90 \cdot 2^8$$
$$and\ S_2 = \sum_{j=1}^{10} j \cdot {}^{10}C_j$$
$$= \sum_{j=1}^{10} j \cdot \frac{10}{j} {}^{9}C_{j-1} = 10 \sum_{j=1}^{10} {}^{9}C_{j-1}$$
$$= 10 \cdot 2^9$$
$$S_3 = \sum_{j=1}^{10} j^{2 10}C_j = 10 \sum_{j=1}^{10} j \cdot {}^{9}C_{j-1}$$
$$= 10 \cdot 11 \cdot 2^{9-1} = 110 \cdot 2^8$$
$$\left[\because \sum r \cdot {}^{n}C_r = n \cdot 2^{n-1}\right]$$
$$= 55 \cdot 2^9$$

Question5

If $y = \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \ldots + \infty$, then

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Options:

A.

$y^2 - 2y + 5 = 0$

B.

$y^2 + 2y - 7 = 0$

C.

$y^2 - 3y + 4 = 0$

D.

$y^2 + 4y - 6 = 0$

Answer: B

Solution:

Given,

$$\begin{aligned} y &= \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \ldots + \infty \\ &= \frac{3}{1!} \cdot \frac{1}{4} + \frac{3 \cdot 5}{2!} \cdot \frac{1}{4^2} + \frac{3 \cdot 5 \cdot 7}{3!} \cdot \frac{1}{4^3} + \ldots + \infty \end{aligned}$$

Since, using binomial expansion

$$(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!} x^2 + \frac{n(n+1)(n+2)}{3!} x^3 + \ldots$$

Here, $nx = \frac{3}{4}$ $\ldots (i)$

$$\frac{n(n+1)}{2} x^2 = \frac{3 \cdot 5}{2!} \cdot \frac{1}{4^2}$$

$$\Rightarrow \quad n(n+1)x^2 = \frac{15}{16} \qquad \ldots (ii)$$

From Eq. (i) and (ii), we get

$$\begin{aligned} \frac{n(n+1)x^2}{n^2x^2} &= \frac{15/16}{(3/4)^2} \\ &= \frac{15/16}{9/16} = \frac{5}{3} \end{aligned}$$

$\Rightarrow \quad \frac{n+1}{n}=\frac{5}{3}$
$\Rightarrow \quad 3n+3=5n$
$\Rightarrow \quad n=\frac{3}{2}$
So, $x=\frac{3}{4}\cdot\frac{1}{n}=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$

$\therefore \quad y=(1-x)^{-n}-1=\left(1-\frac{1}{2}\right)^{-3/2}-1$

$=\left(\frac{1}{2}\right)^{-3/2}-1=2^{3/2}-1=2\sqrt{2}-1$

So, $\quad y+1=2\sqrt{2}-1+1=2\sqrt{2}$
$(y+1)^2=(2\sqrt{2})^2=8$
$\Rightarrow y^2+2y+1-8=0$
$\Rightarrow \quad y^2+2y-7=0$
Thus, the quadratic equation is
$y^2+2y-7=0$

Question6

Sum of the coefficients of x^4 and x^6 in the expansion of $\left(1+x-x^2\right)^6$ is

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Options:

A.

121

B.

-91

C.

11

D.

31

Answer: C

Solution:

We will expand $\left(1+x-x^2\right)^6$ and find the coefficients of x^4 and x^6.

Multimonial expansion states that

$$(a_1+a_2+a_3)^n = \sum \frac{n!}{k_1!k_2!k_3!} a_1^{k}a_2^{k_2}a_3^{k_3}$$

where $k_1+k_2+k_3=n$

In our case, $a_1=1, a_2=x, a_3=-x^2$ and $n=6$

From $k_1+k_2+k_3=6$

$\Rightarrow \quad k_1=6-k_2-k_3 \qquad \ldots(i)$

To get x^4, we can have different combinations of x and x^2

k_1 : number of times 1 is chosen

k_2 : number of times x is chosen

k_3 : number of times x^2 is chosen

The equation is: $k_2+2k_3=4 \qquad \ldots(ii)$

Now, if $k_3=0$, then $k_2=4$ and $k_1=2$

If $k_3=1$, then $k_2=2$ and $k_1=3$

If $k_3=2$, then, $k_2=0, k_1=4$

Now, find the coefficients for each valid combination.

Coefficient of

$$x^4 = \frac{6!}{2!4!0!} - \frac{6!}{3!2!1!} + \frac{6!}{4!0!2!}$$
$$= 15-60+15=-30$$

Now, for x^6, $k_2+2k_3=6$ and

$k_1+k_2+k_3=6$

If $k_3=0$, then $k_2=6$, and $k_1=0$

If $k_3=1$, then $k_2=4$, and $k_1=1$

If $k_3=2$ then $k_2=2$ and $k_1=2$

If $k_3=3$, then $k_2=0$, and $k_1=3$

So, coefficient of $x^6 = \frac{6!}{0!6!0!} - \frac{6!}{1!4!1!} + \frac{6!}{2!2!2!} - \frac{6!}{3!0!3!}$

$=1-30+90-20=41$

$\therefore$ Sum of the coefficient of x^4 and $x^6 = 41-30=11$

Question7

If $11^{12}-11^2 = k\left(5\times10^9+6\times10^9+33\times10^8+110\times10^7+\ldots+33\right)$, then $k=$

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Options:

A.

20

B.

50

C.

100

D.

200

Answer: D

Solution:

Given,

$11^{12}-11^{2}=k\left(5\times10^{9}+6\times10^{9}+33\times10^{8}+110\times10^{7}+\ldots+33\right)$

$11^{12}-11^{2}=(10+1)^{12}-(10+1)^{2}$

$=10^{12}+12\times10^{11}+66\times10^{10}+220\times10^{9}\cdots+66\times10^{2}+120+1-121$

$=10^{12}+12\times10^{11}+66\times10^{10}+220\times10^{9}\ldots+66\times10^{2}$

$=2\times10^{2}\left(5\times10^{9}+6\times10^{9}+33\times10^{8}+110\times10^{7}+\ldots+33\right)$

$=200\left(5\times10^{9}+6\times10^{9}+33\times10^{8}+110\times10^{7}+\ldots+33\right)$

$\therefore k=200$

Question8

If $C_0, C_2, \ldots, C_n$ are the binomial coefficients in the expansion of $(1+x)^n$, then

$$(C_0+C_1)-(C_2+C_3)+(C_4+C_5)-(C_6+C_7)+\ldots=$$

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Options:

A.

$2^{n/2}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)$

B.

$2^{n/2}\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}\right)$

C.

$2^{n/2}\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}\right)$

D.

$2^{n/2}\left(\cos\frac{n\pi}{4}+\sin\frac{n\pi}{4}\right)$

Answer: D

Solution:

$\because (1+x)^n=\sum_{k=0}^{n} C_k x^k$

and

$(C_0+C_1)-(C_2+C_3)+(C_4+C_5)-\cdots$

can be grouped as

$$\sum_{k=0}^{n/2}(-1)^k\left(C_{2k}+C_{2k+1}\right)$$
$$\text{and }(1+i)^n=\sum_{k=0}^{n} C_k i^k$$

$\operatorname{Re}\left[(1+i)^n\right]=C_0-C_2+C_4-C_6+\cdots$
$\operatorname{Im}\left[(1+i)^n\right]=C_1-C_3+C_5-C_7+\cdots$

Adding both

$\operatorname{Re}\left[(1+i)^n\right]+\operatorname{Im}\left[(1+i)^n\right]$
$=(C_0+C_1)-(C_2+C_3)+(C_4+C_5)-\ldots$

Now, $(1+i)^n$

$=(\sqrt{2})^n\left(\cos\left(\frac{n\pi}{4}\right)+i\sin\left(\frac{n\pi}{4}\right)\right)$

Hence, $(C_0+C_1)-(C_2+C_3)+(C_4+C_5)$

$-\ldots=\operatorname{Re}\left[(1+i)^n\right]+\operatorname{Im}\left[(1+i)^n\right]$
$$=2^{n/2}\left(\cos\left(\frac{n\pi}{4}\right)+\sin\left(\frac{n\pi}{4}\right)\right)$$

Question9

The mean and variance of a binomial distribution are x and 5 respectively. If x is an integer, then the possible values for x are

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Options:

A.

$6, 10, 30$

B.

$8, 12, 28$

C.

$10, 15, 25$

D.

$9, 18, 24$

Answer: A

Solution:

The mean of a binomial distribution is x, and the variance is 5.

We know that the mean, $\mu = np = x$. This means n times p is x.

The formula for variance is $\sigma^2 = np(1-p) = 5$. This means n times p times $(1-p)$ is 5.

From the mean formula, $p = \frac{x}{n}$.

Substitute $p = \frac{x}{n}$ into the variance formula:

$x\left(1 - \frac{x}{n}\right) = 5$

If we open the bracket, we get:$x - \frac{x^2}{n} = 5$

Take $\frac{x^2}{n}$ to the other side:$\frac{x^2}{n} = x - 5$

Now, solve for n:$n = \frac{x^2}{x-5}$

To get a whole number for n, $x - 5$ must divide x^2 exactly. Let's check different x values:

If $x = 6$:$n = \frac{36}{1} = 3636$ is an integer.

If $x = 10$:$n = \frac{100}{5} = 2020$ is an integer.

If $x = 30$:$n = \frac{900}{25} = 3636$ is an integer.

So, the possible values for x are: $6, 10, 30$.

Question10

If the coefficients of x^{10} and x^{11} in the expansion of $\left(1+\alpha x+\beta x^2\right)(1+x)^{11}$ are 396 and 144 respectively, then $\alpha^2+\beta^2=$

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Options:

A.

10

B.

13

C.

25

D.

20

Answer: B

Solution:

As we know that, coefficient of x^{11} in the expansion of $(1+x)^{11}$ is ${}^{11}C_{11}$.

Now, we have to find coefficient of x^{11} in the expansion of $\left(1+\alpha x+\beta x^2\right)(1+x)^{11}$. When we choosing 1 from $\left(1+\alpha x+\beta x^2\right)$ then we need x^{11} from $(1+x)^{11}$.

So, coefficient $= {}^{11}C_{11}=1$

Similarly, we choosing αx from $\left(1+\alpha x+\beta x^2\right)$ then we need x^{10} from $(1+x)^{11}=$ Coefficient $=\alpha \times {}^{11}C_{10}=11\alpha$ and when we choosing βx^2 from $\left(1+\alpha x+\beta x^2\right)$ then we need x^9 from $(1+x)=$ Coefficient $=\beta \times {}^{11}C_9=55 \cdot \beta$

$\therefore$ Coefficient of x^{11} in the expansion of $\left(1+\alpha x+\beta x^2\right)$

$(1+x)^{11}=1+11\alpha+55\beta=144$ (given) ...(i)

Just, in similar way, we have to find coefficient of x^{10} such that

$11+55\alpha+165\beta=396$ $\quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get;

$$\alpha=-2 \text{ and } \beta=3$$
$$\therefore \alpha^2+\beta^2=(-2)^2+(3)^2=13$$

Question11

If $-\frac{2}{3}<x<\frac{2}{3}$, then the value of the 5 th term in the expansion of $\frac{1}{\sqrt[3]{2-3x}}$ when $x=\frac{1}{2}$ is

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Options:

A.

$\frac{35}{256(\sqrt[3]{2})}$

B.

$\frac{35}{768(\sqrt[3]{2})}$

C.

$\frac{7}{768(\sqrt[3]{2})}$

D.

$\frac{105}{256(\sqrt[3]{2})}$

Answer: B

Solution:

Given, $\frac{-2}{3}<x<\frac{2}{3}$

$\Rightarrow -2<3x<2 \Rightarrow -1<\frac{3x}{2}<1$

Now, $\frac{1}{3\sqrt{2-3x}}=\frac{1}{(2-3x)^{1/3}}$

$=\frac{1}{2^{1/3}}\left(1-\frac{3x}{2}\right)^{-1/3}$

$=\frac{1}{3\sqrt{2}}\left[1+\left(\frac{1}{3}\right)\left(\frac{3x}{2}\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}+1\right)}{2\times1}\left(\frac{3x}{2}\right)^2+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}+1\right)\left(\frac{1}{3}+2\right)}{3\times2\times1}\left(\frac{3x}{2}\right)^3+\frac{\frac{1}{3}\left(\frac{1}{3}+1\right)\left(\frac{1}{3}+2\right)\left(\frac{1}{3}+3\right)}{4\times3\times2\times1}\left(\frac{3x}{2}\right)^4+\ldots\right]$

$\therefore$ 5 th term when $x=\dfrac{1}{2}$ is given by

$$\frac{1}{\sqrt[3]{2}}\left[\frac{1}{3}\times\frac{4}{3}\times\frac{7}{3}\times\frac{10}{3}\times\frac{1}{24}\times\frac{81}{16}\times\frac{1}{16}\right]$$
$$=\frac{1}{3\sqrt{2}}\left[\frac{7\times5}{24\times2\times6}\right]=\frac{35}{768(\sqrt[3]{2})}$$

Question12

The terms containing $x^r y^s$ (for certain r and s) are present in both the expansions of $\left(x+y^2\right)^{13}$ and $\left(x^2+y\right)^{14}$. If α is the number of such terms, then the sum $\alpha\sum_{r,s}(r+s)=$

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Options:

A.

27

B.

40

C.

18

D.

35

Answer: C

Solution:

General term for $\left(x+y^2\right)^{13}$

$={}^{13}C_n x^{13-n} y^{2n}$

The general term for $\left(x^2+y\right)^{14}$

$={}^{14}C_{r_2} x^{2r_2} y^{14-r_2}$

Equate the power of x and y

$13-r_1=2r_2$
$2r_1=14-r_2$

Solving these equation, we get

$r_1 = 5, r_2 = 4$

The common term is $x^{13-5} y^{2(9)} = x^8 y^{10}$ The common term is $x^8 y^{10}$

$\Rightarrow \quad r = 8, s = 10$

$\alpha = 1$, since there is only one common term

$$\Rightarrow \quad \alpha(r+s) = 1(8+10) = 18$$
$$\therefore \text{ Sum } \alpha \sum_{r,s} (r+s) = 18$$

Question13

The coefficient of x^3 in the power series expansion of $\frac{1+4x-3x^2}{(1+3x)^3}$ is

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Options:

A.

-27

B.

27

C.

153

D.

-153

Answer: A

Solution:

$\frac{1+4x-3x^2}{(1+3x)^3} = \left(1+4x-3x^2\right)(1+3x)^{-3}$

$$\Rightarrow (1+3x)^{-3} = 1 - 3(3x) + \frac{(-3)(-4)(3x)^2}{2!} + \frac{(-3)(-4)(-5)(3x)^3}{3!} + \ldots$$

$$\Rightarrow (1+3x)^{-3} = 1 - 9x + 54x^2 - 270x^3 + \ldots$$

$$\begin{aligned}(1+4x-3x^2)&\left(1-9x+54x^2-270x^3+\ldots\right)\\ &= 1\left(-270x^3\right) + 4x\left(5x^2\right) - 3x^2(-9x)\\ &= -270x^3 + 216x^3 + 27x^3\\ &= -270 + 216 + 27 = -27\end{aligned}$$

$\Rightarrow$ The coefficient of $x^3 = -27$

Question14

If k is a positive integer and 10^k is a divisor of the number $9^{11} + 11^9$, then the greatest value of k is

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Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

$9^{11} + 11^9 = (10-1)^{11} + (10+1)^9$

$$= {}^{11}C_0 10^{11} + {}^{11}C_1 10^9 + \ldots {}^{11}C_{11} + {}^{9}C_0 10^{10} + {}^{9}C_1 10^8 + \ldots + {}^{9}C_9$$

Sum of last two terms of both expansion as other term have at least 10^2 as a factor

$(110-1)+90+1=200$

$\therefore \quad 9^{11}+11^{9}=$ Multiple of

$100+200=$ Multiple of 100

Since $200=2 \times 10^{2}$

So, $9^{11}+11^{9}$ is divisible by 10^{2}

$\therefore \quad k=2$

Question15

The number of all possible values of k for which the expansion $(\sqrt{x}+\sqrt[k]{y})^{10}$ will have exactly nine irrational terms is

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Options:

A.

3

B.

4

C.

5

D.

6

Answer: C

Solution:

Given expansion $\left(x^{\frac{1}{2}}+y^{\frac{1}{k}}\right)^{10}$

General term $={ }^{10} C_{r} x^{\frac{10-r}{2}} y^{\frac{r}{k}}$

Given expansion have only 9 irrational term

$\therefore$ Expansion has only two rational term Two rational term is possible

LCM of 2 and k is 10

$\therefore k=5$

Question16

Coefficient of x^2 in the expansion of $\left(x^2+x-2\right)^5$ is

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Options:

A.

800

B.

756

C.

0

D.

512

Answer: C

Solution:

$\left(x^2+x-2\right)^5$

Apply multinomial theorem

$\Sigma \frac{5!}{p!q!r!}\left(x^2\right)^p(x)^q(-2)^r$

Such that $p+q+r=5$ and p, q, r are non-negative integer.

$2p+q=2$
$(p, q, r)=(1,0,4)(0,2,3)$
$\frac{5!}{4!}\left(-4^4+\frac{5!}{2!3!}(-2^3\right.$
$5 \times 16+10(-8)=80-80=0$

Question17

If P_n denotes the product of the binomial coefficients in the expansion of $(1+x)^n$, then $\frac{P_{n+1}}{P_n}=$

Options:

A.

$\frac{n+1}{n!}$

B.

$\frac{n^n}{n!}$

C.

$\frac{(n+1)^n}{(n+1)!}$

D.

$\frac{(n+1)^{n+1}}{(n+1)!}$

Answer: D

Solution:

We have, $P_n = {}^nC_0 \cdot {}^nC_1 {}^nC_2 \ldots {}^nC_n$ and

$$P_{n+1} = {}^{n+1}C_0 \cdot {}^{n+1}C_1 {}^{n+1}C_2 \cdots {}^{n+1}C_{n+1}$$
$$\therefore \frac{P_{n+1}}{P_n} = \frac{{}^{n+1}C_0 \cdot {}^{n+1}C_1 {}^{n+1}C_2 \cdots {}^{n+1}C_{n+1}}{{}^nC_0 \cdot {}^nC_1 \cdot {}^nC_2 \cdots {}^nC_n}$$
$$= \left(\frac{{}^{n+1}C_1}{{}^nC_0}\right)\left(\frac{{}^{n+1}C_2}{{}^nC_1}\right)\left(\frac{{}^{n+1}C_3}{{}^nC_2}\right)\cdots\left(\frac{{}^{n+1}C_{n+1}}{{}^nC_n}\right)$$
$$= \left(\frac{n+1}{1}\right)\left(\frac{n+1}{2}\right)\left(\frac{n+1}{3}\right)\cdots\left(\frac{n+1}{n+1}\right)$$
$$= \frac{(n+1)^{n+1}}{(n+1)!}$$

Question18

The coefficient of x^3 in the expansion of $\frac{x^4+1}{(x^2+1)(x-1)}$ when it is expressed in terms of positive integral powers of x, is

Options:

A.

0

B.

1

C.

16

D.

24

Answer: A

Solution:

Given, $\frac{x^4+1}{(x^2+1)(x-1)}$

$$= \frac{x^4\left(1+\frac{1}{x^4}\right)}{x^3\left(1+\frac{1}{x^2}\right)\left(1-\frac{1}{x}\right)}$$
$$= x\left(1+\frac{1}{x^4}\right)\left(1+\frac{1}{x^2}\right)^{-1}\left(1-\frac{1}{x}\right)^{-1}$$
$$= x\left(1+\frac{1}{x^4}\right)\left(1-\frac{1}{x^2}\right)\left(1+\frac{1}{x}+\frac{1}{x^2}+\ldots\right)$$

Clearly, there is no terms containing x^3 coefficient of $x^3 = 0$

Question19

If $(1+x)^n = \sum_{r=0}^{n} C, x^r$, then the value of $C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + \ldots + (C_0 + C_1 + C_2 + \ldots + C_n)$ is

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Options:

A.

nR^{n-1}

B.

$2^n + n$

C.

$(n+2)2^n$

D.

$(n+2)2^{n-1}$

Answer: D

Solution:

The given sum can be rewritten as

$\sum_{k=0}^{n} \sum_{r=0}^{k} C_r$

Now, changing order of summarion

$\sum_{r=0}^{n} \sum_{k=r}^{n} C_r \quad [\because \sum_{r=0}^{n} C_r = 2^n]$

$$\sum_{r=0}^{n} C_r \sum_{k=r}^{n} 1 = \sum_{r=0}^{n} C_r(n-r+1)$$

$$\begin{aligned} &\Rightarrow (n+1)\sum_{r=0}^{n} C_r - \sum_{r=0}^{n} r \cdot C_r \\ &= (n+1)2^n - \sum_{r=0}^{n} r \cdot {}^{n-1}C_{r-1} \times \frac{n}{r} \\ &= (n+1)2^n - n\sum_{r=0}^{n} n-1C_r \\ &= (n+1)2^n - n \cdot 2^{n-1} \\ &= n \cdot 2^n + 2^n - n \cdot 2^{n-1} \\ &= 2^{n-1}(n+2) \end{aligned}$$

Question20

If x is so large that terms containing $x^{-3}, x^{-4}, x^{-5}, \ldots$ can be neglected, then the approximate value of $\left(\frac{3x-5}{4x^2+3}\right)^{-1/5}$ is

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Options:

A.

$\left(\frac{3}{4x}\right)^{4/5}\left(1-\frac{4}{3x}-\frac{7}{5x^2}\right)$

B.

$\left(\frac{4x}{3}\right)^{4/5}\left(1+\frac{4}{3x}+\frac{13}{5x^2}\right)$

C.

$\left(\frac{4x}{3}\right)^{4/5}\left(1+\frac{4}{3x}-\frac{13}{5x^2}\right)$

D.

$\left(\frac{3}{4x}\right)^{4/5}\left(1-\frac{4}{3x}+\frac{7}{5x^2}\right)$

Answer: B

Solution:

$$\text{We have, } \left(\frac{3x-5}{4x^2+3}\right)^{-\frac{4}{5}}=\left(\frac{4x^2+3}{3x-5}\right)^{\frac{4}{5}}$$

$$=\left\{\frac{4x^2\left(1+\frac{3}{4x^2}\right)}{3x\left(1-\frac{5}{3x}\right)}\right\}^{\frac{4}{5}}=\left(\frac{4x}{3}\right)^{\frac{4}{5}}\frac{\left(1+\frac{3}{4x^2}\right)^{4/5}}{\left(1-\frac{5}{3x}\right)^{4/5}}$$

$$\Rightarrow\left(\frac{4x}{3}\right)^{\frac{4}{5}}\left(1+\frac{3}{4x^2}\right)^{\frac{4}{5}}\cdot\left(1-\frac{5}{3x}\right)^{-\frac{4}{5}}$$

$$=\left(\frac{4x}{3}\right)^{\frac{4}{5}}\left(1+\frac{4}{5}\cdot\frac{3}{4x^2}+\ldots\right)$$

$$\left(1+\frac{5}{3x}\cdot\frac{4}{5}+\frac{4}{5}\left(+\frac{9}{5}\right)\left(\frac{5}{3x}\right)\right)^2\frac{1}{2!}+\ldots\ldots$$

$$=\left(\frac{4x}{3}\right)^{\frac{4}{5}}\left(1+\frac{3}{5x^2}+\ldots\right)\left(1+\frac{4}{3x}+\frac{2}{x^2}\right)$$

$$=\left(\frac{4x}{3}\right)^{\frac{4}{5}}\left(1+\frac{4}{3x}+\left(\frac{3}{5}+2\right)^{\frac{1}{x^2}}\right)$$

$$=\left(\frac{4x}{3}\right)^{\frac{4}{5}}\left(1+\frac{4}{3x}+\frac{13}{5x^2}\right)$$

Question21

The independent term in the expansion of $\left(1+x+2x^2\right)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ is

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Options:

A. $\frac{18}{7}$

B. $\frac{7}{18}$

C. $-\frac{7}{18}$

D. $-\frac{18}{7}$

Answer: B

Solution:

We have, $(1+x+2x^2)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$

$= (1+x+2x^2)$
$\left\{\sum_{r=0}^{9} {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r\right\}$

$= (1+x+2x^2)$
$\left\{\sum_{r=0}^{9} {}^9C_r\left(\frac{3}{2}\right)^{9-r} x^{18-2r}\left(-\frac{1}{3}\right)^1 x^{-r}\right\}$

$= (1+x+2x^2)$
$\left\{\sum_{r=0}^{9} {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}\right\}$

$$= \sum_{r=0}^{9} {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r} + \sum_{r=0}^{9} {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{19-3r} + 2\left\{\sum_{r=0}^{9} {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{20-3r}\right\}$$

Clearly, first term independent of x by

$18-3r=0 \Rightarrow r=6$

$\therefore$ Coefficient of independent of x is

$${}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6 = {}^9C_3 \times \frac{3^3}{8} \times \frac{1}{3^6}$$
$$= \frac{9\times 8\times 7}{3\times 2} \times \frac{1}{8} \times \frac{1}{3^3} = \frac{7}{18}.$$

Question22

For $|x| < \frac{1}{\sqrt{2}}$, the coefficient of x in the expansion of $\frac{(1-4x)^2(1-2x^2)^{1/2}}{(4-x)^{3/2}}$ is

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Options:

A. $\frac{61}{64}$

B. $-\frac{61}{64}$

C. $\frac{69}{64}$

D. $-\frac{69}{64}$

Answer: B

Solution:

To find the coefficient of x in the expansion of

$\frac{(1-4x)^2(1-2x^2)^{\frac{1}{2}}}{(4-x)^{\frac{3}{2}}}$

we proceed by approximating each component in the product.

Expansion of $(1-4x)^2$:

$(1-4x)^2 = 1 - 8x + 16x^2$

Approximation of $\left(1-2x^2\right)^{\frac{1}{2}}$:

Using the binomial approximation for small x^2, we have:

$\left(1-2x^2\right)^{\frac{1}{2}} \approx 1 - x^2$

Approximation of $\left(1-\frac{x}{4}\right)^{-\frac{3}{2}}$:

Again, using binomial expansion, we write:

$\left(1-\frac{x}{4}\right)^{-\frac{3}{2}} \approx 1 + \frac{3}{2} \times \frac{x}{4} = 1 + \frac{3x}{8}$

Putting it all together, we compute the product:

$\frac{1}{8}\left[(1-8x+16x^2)(1-x^2)(1+\frac{3x}{8})\right]$

Next, we multiply these expressions and collect the terms involving x:

$$= \frac{1}{8}\left[(1-8x+16x^2)(1-x^2)(1+\frac{3x}{8})\right]$$
$$= \frac{1}{8}\left[(1-8x+16x^2)(1-x^2+\frac{3x}{8}-\frac{3x^3}{8}+\cdots)\right]$$

The coefficient of x from $(1-8x+16x^2)(1)$ is -8.

The contribution due to combining 1 from $(1-8x+16x^2)$ with the $\frac{3x}{8}$ term from $(1+\frac{3x}{8})$ is $\frac{3}{8}$.

Thus, the total coefficient of x from this product is:

$\frac{1}{8}\left(\frac{3}{8}-8\right)$

Simplifying, we calculate:

$\frac{1}{8}\cdot\left(-\frac{61}{8}\right)=-\frac{61}{64}$

Therefore, the coefficient of x is $-\frac{61}{64}$.

Question23

If P is the greatest divisor of $49^n+16n-1$ for all $n \in N$, then the number of factors of P is

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Options:

A. 12

B. 15

C. 7

D. 13

Answer: C

Solution:

Given that

P is the greatest divisor of $49^n+16n-1$, for $\forall$ all $n \in N$.

$$
\begin{aligned}
& 49^n+16n-1=(1+48)^n+16n-1 \\
= & 1+n\cdot 48+\frac{n(n-1)}{2}\cdot(48)^2+\ldots..16n-1 \\
= & 64n+{}^nC_2(48)^2+{}^nC_3(48)^3+\ldots(48)^n \\
= & 64\left(n+{}^nC_2 6^2+{}^nC_3 6^3\cdot 8+\ldots 6^n\cdot 8^{n-2}\right)
\end{aligned}
$$

So, clearly it is divisible by 64 .

Now, number factor of 64 is

$64=2^6=(6+1)=7$

Question24

If the coefficients of r th, $(r+1)$ th and $(r+2)$ th terms in the expansion of $(1+x)^n$ are in the ratio of $4:15:42$, then $n-r$ is equal to

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Options:

A. 18

B. 15

C. 14

D. 17

Answer: C

Solution:

Given that coefficient of r th: $(r+1)$ th : $(r+2)$ th $=4:15:42$ in expansion of $(1+x)^n$.

$$\Rightarrow {}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 4:15:42 = K$$

$$\Rightarrow \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{4K}{15K} \Rightarrow \frac{r}{n-r+1} = \frac{4}{15} \qquad \ldots(i)$$

$$\text{and } \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15K}{42K} \Rightarrow n-r = \frac{14(r+1)}{5} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{5r}{14r+19} = \frac{4}{15}$$

$$\Rightarrow 75r = 56r + 76 \Rightarrow 19r = 76$$

$$\Rightarrow r = 4$$

$$\Rightarrow n-r = \frac{14 \times (4+1)}{5} = 14 \text{ [from Eq. (i)}$$

Question25

If the coefficients of $(2r+6)$ th and $(r-1)$ th terms in the expansion of $(1+x)^{21}$ are equal, then the value of r is equal to

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Options:

A. 7

B. 5

C. 6

D. 8

Answer: C

Solution:

Given that coefficients of $(2r+6)$ th and $(r-1)$ th terms in the expansion of $(1+x)^{21}$ are equal.

$$\begin{aligned} T_{2r+5+1} &= {}^{21}C_{2r+5}(1)^{21-2r-5}x^{2r+5} \\ T_{r-2+1} &= {}^{21}C_{r-2}(l)^{21-r+2}x^{r-2} \\ \because \quad {}^{21}C_{2r+5} &= {}^{21}C_{r-2} \\ \Rightarrow \quad 2r+5 &= r-2 \\ r &= 3 \qquad\qquad (not\ possible) \end{aligned}$$

So, $\quad 2r+5 = 21-(r-2)$

$$[\because {}^nC_r = {}^nC_{n-r}]$$

$\Rightarrow \quad 2r+5 = 21-r+2$

$\Rightarrow \quad 3r = 18$

$\Rightarrow \quad r = 6$

Question26

If the $2\text{nd}, 3\text{rd}$ and 4 th terms in the expansion of $(x+a)^n$ are $96, 216, 216$ respectively and n is a positive integer, then $a+x=$

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Options:

A. $n+1$

B. n

C. $n-1$

D. $\frac{n}{2}$

Answer: A

Solution:

We know that r th term in the expansion $(x+a)^n$ is given by

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} given,\ T_2 &= {}^nC_1 x^{n-1} a = 96 \quad \ldots(i) \\ T_3 &= {}^nC_2 x^{n-2} a^2 = 216 \quad \ldots(ii) \\ T_4 &= {}^nC_3 x^{n-3} a^3 = 216 \quad \ldots(iii) \end{aligned}$$

On dividing Eq. (ii) from Eq. (i), we get

$$\frac{{}^nC_2 x^{n-2} a^2}{{}^nC_1 x^{n-1} a} = \frac{216}{96}$$

$$\frac{\frac{n!}{(n-2)!2!} x^{n-1-1} a^2}{\frac{n!}{(n-1)!1!} x^{n-1} a} = \frac{9}{4}$$

$$\frac{1}{2} \cdot \frac{(n-1)(n-2)!}{(n-2)!} \frac{a}{x} = \frac{9}{4}$$

$$(n-1)\frac{a}{x} = \frac{9}{2}$$

$$(n-1)a = \frac{9}{2}x \quad \ldots(iv)$$

Similarly, on dividing Eq. (iii) from Eq. (ii), we get

$$\frac{{}^nC_3 x^{n-3} a^3}{{}^nC_2 x^{n-2} a^2} = \frac{216}{216}$$

$$\Rightarrow \frac{\frac{n!}{(n-3)!3!} x^{n-2-1} a^3}{\frac{n!}{(n-2)!2!} x^{n-2} a^2} = 1$$

$$\Rightarrow \frac{2 \cdot 1(n-2)(n-3)!a}{3 \cdot 2 \cdot l(n-3)!} \frac{a}{x} = 1$$

$$\Rightarrow \quad (n-2)a = 3x \quad \ldots(v)$$

Now, on dividing Eq. (v) from Eq. (iv) we get,

$$\frac{(n-2)a}{(n-1)a} = \frac{3x}{\frac{9}{2}x} = \frac{2}{3}$$

$$\Rightarrow \quad 3n - 6 = 2n - 2$$

$$n = 4$$

From Eq. (v), we get

$$(4-2)a = 3x$$

$$a = \frac{3}{2}x \quad \ldots(vi)$$

From Eq (i), we get

$${}^4C_1 x^{4-1} a^1 = 96$$

$$\frac{4!}{3!1!} x^3 a = 96$$

$$4x^3 \cdot \frac{3}{2}x = 96 \quad [From Eq.\,(vi)]$$

$$x^4 = \frac{96}{6} = 16$$

$$x = 2$$

Using Eq. (vi), we get

$a = 3$

$Clearly, \quad x + a = 2 + 3 = 4 + 1$

$x + a = n + 1$

Question27

If $|x| < 1$, then the number of terms in the expansion of $\left[\frac{1}{2}\left(1 \cdot 2 + 2 \cdot 3x + 3 \cdot 4x^2 + \ldots . \infty\right)\right]^{-25}$

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Options:

A. Infinite

B. 101

C. 76

D. 51

Answer: C

Solution:

Given the expression:

$\frac{1}{2}\left(1 \cdot 2 + 2 \cdot 3x + 3 \cdot 4x^2 + \ldots + \infty\right)^{-25}$

First, consider the series inside the parentheses:

$1 \cdot 2 + 2 \cdot 3x + 3 \cdot 4x^2 + \ldots + \infty$

This can be rewritten as:

$2\left(1 + 3x + 6x^2 + \ldots + \infty\right)$

Recognizing the series as a result of differentiating and manipulating a geometric series, it sums to:

$2 \frac{1}{(1-x)^3}$

So,

$\frac{1}{2} \times 2 \times \frac{1}{(1-x)^3} = \frac{1}{(1-x)^3}$

Thus, the expression evaluates to:

$\frac{1}{2}\left(1 \cdot 2 + 2 \cdot 3x + 3 \cdot 4x^2 + \ldots + \infty\right)^{-25} = \left(\frac{1}{(1-x)^3}\right)^{-25} = (1-x)^{75}$

Therefore, the number of terms in the expansion of $(1-x)^{75}$ is:

$75 + 1 = 76$

Question28

If the ratio of the terms equidistant from the middle term in the expansion of $(l+x)^{12}$ is $\frac{1}{256}(x \in N)$, then sum of all the terms of the expansion $(1+x)^{12}$ is

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Options:

A. 4^{12} or 6^{12}

B. 3^{12} or 5^{12}

C. 6^{12} or 7^{12}

D. 12^{12}

Answer: B

Solution:

In the expansion of $(1+x)^{12}$, we need to determine the sum of all its terms given the ratio of terms equidistant from the middle is $\frac{1}{256}$.

Let's find the middle term and the equidistant terms:

The k-th term is denoted by:

$T_k = \binom{12}{k} x^k$

The equidistant terms from the middle are:

$T_{12-k} = \binom{12}{12-k} x^{12-k}$

Since, the ratio of equidistant terms is given by:

$\frac{T_k}{T_{12-k}} = \frac{\binom{12}{k} x^k}{\binom{12}{12-k} x^{12-k}} = \frac{1}{256}$

Since $\binom{12}{12-k} = \binom{12}{k}$, the equation simplifies to:

$\frac{x^k}{x^{12-k}} = x^{-12+2k} = \frac{1}{256} = 2^{-8}$

This gives us:

$-12+2k = -8$

Solving for k, we find:

$2k = 4 \implies k = 2$

Thus, the equidistant terms are T_2 and T_{10}.

Now, calculate the sum of all terms in the expansion:

When $x = 2$:

$S = (1+2)^{12} = 3^{12}$

When $x = 4$:

$S = (1+4)^{12} = 5^{12}$

Therefore, the sum of all terms could either be 3^{12} or 5^{12} based on the value of x.

Question29

If the eleventh term in the binomial expansion of $(x+a)^{15}$ is the geometric mean of the eighth and twelfth terms, then the greatest term in the expansion is

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Options:

A. 7 th term

B. 8 th term

C. 9 th term

D. 10 th term

Answer: B

Solution:

From the given condition, one g^6

$$\left({}^{15}C_{10}x^2a^{10}\right)^2 = \left({}^{15}C_7x^8a^7\right)\left({}^{15}C_{11}x^4a^{11}\right)$$

$$\Rightarrow \frac{a}{x} = \sqrt{\frac{75}{77}}$$

Now, $(x+a)^{15} = x^{15}\left(1+\frac{a}{x}\right)^{15}$

Consider the expansion of $\left(1+\frac{a}{x}\right)^{15}$

$$T_r = T_{r+1} \Rightarrow \frac{r}{(n-r+1)\left(\frac{a}{x}\right)} = 1$$

$$\Rightarrow r = 16\sqrt{\frac{75}{77}} - r\sqrt{\frac{75}{77}} \Rightarrow r = \frac{16\sqrt{\frac{75}{77}}}{1+\sqrt{\frac{75}{77}}}$$

$r = 7$

If $r=7 \Rightarrow T_7 < T_8$ and $r=8 \Rightarrow T_8 > T_9$ Hence, T_8 is the greatest term.

Question30

The sum of the rational terms in the binomial expansion of $\left(\sqrt{2}+3^{1/5}\right)^{10}$ is

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Options:

A. 41

B. 39

C. 32

D. 30

Answer: A

Solution:

$(r+1)$ th term in the given expansion is given by

$t_{r+1} = {}^{10}C_r(2)^{\frac{10-r}{2}} \cdot 3^{\frac{r}{5}}$

where $r = 0, 1, 2, \ldots 10$

For rational terms

$r = a$ multiple of $5 = 0, 5, 10$

$10 - r = a$ multiple of $2 = 0, 2, 4$

From Eqs. (i) and (ii) possible values of are 0 and 10

$\therefore$ Sum of rational terms

$= t_1 + t_{11} = {}^{10}C_0(\sqrt{2})^{10}$
$\left(3^{\frac{1}{5}}\right)^0 + {}^{10}C_{10}(\sqrt{2})^0\left(3^{\frac{1}{5}}\right)^{10}$
$= 2^5 + 3^2 = 32 + 9 = 41$

Question31

If the coefficients of x^5 and x^6 are equal in the expansion of $\left(a+\frac{x}{5}\right)^{65}$, then the coefficient of x^2 in the expansion of $\left(a+\frac{x}{5}\right)^4$ is.

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Options:

A. 1

B. $\frac{32}{25}$

C. 2

D. $\frac{24}{25}$

Answer: D

Solution:

Coefficient of x^5 in $\left(a+\frac{x}{5}\right)^{65}$ is

${}^{65}C_5 \cdot a^{60} \cdot \left(\frac{1}{5}\right)^5$

and coefficient of x^6 in $\left(a+\frac{x}{5}\right)^{65}$ is

${}^{65}C_6 a^{59}\left(\frac{1}{5}\right)^6$

Now, coefficient of x^5 = Coefficient of x^6 in expansion $\left(a+\frac{x}{5}\right)^{65}$

$$
\begin{aligned}
&{}^{65}C_5 \cdot a^{60} \cdot \left(\frac{1}{5}\right)^5 = {}^{65}C_6 \cdot a^{59}\left(\frac{1}{5}\right)^6 \\
&\Rightarrow \frac{65!}{5!60!} \cdot a^{60} \cdot \frac{1}{5^5} = \frac{65!}{6!59!} a^{59} \cdot \frac{1}{5^6} \\
&\Rightarrow \frac{a}{60} = \frac{1}{6} \times \frac{1}{5} \Rightarrow a = \frac{60}{30} = 2 \\
&\Rightarrow a = 2
\end{aligned}
$$

$\therefore$ Coefficient of x^2 in expansion of $\left(2+\frac{x}{5}\right)^4$ is

$$
\begin{aligned}
&= {}^4C_2 \cdot (2)^2 \cdot \left(\frac{1}{5}\right)^2 \\
&= \frac{4!}{2!2!} \cdot \frac{4}{25} = \frac{24}{25}
\end{aligned}
$$

Question32

If $|x| < \frac{2}{3}$, then the 4th term in the expansion of $(3x-2)^{\frac{2}{3}}$ is :

Options:

A. $\frac{\sqrt[3]{4}}{6} x^3$

B. $-\frac{\sqrt[3]{4}}{6} x^3$

C. $\frac{\sqrt[3]{4}}{8} x^3$

D. $-\frac{\sqrt[3]{4}}{8} x^3$

Answer: B

Solution:

Given the expression $(3x-2)^{\frac{2}{3}}$, we can rewrite it as:

$(3x-2)^{\frac{2}{3}}=(-2+3x)^{\frac{2}{3}}$

This can be further expressed as:

$=(2)^{\frac{2}{3}}\left(-1+\frac{3}{2}x\right)^{\frac{2}{3}}$

To find the 4th term of the expansion of $2^{\frac{2}{3}}\left(-1+\frac{3}{2}x\right)^{\frac{2}{3}}$, we use the formula for the binomial expansion term, which is given by:

$T_4=2^{\frac{2}{3}}\cdot\binom{\frac{2}{3}}{3}\cdot(-1)^{\frac{2}{3}-3}\cdot\left(\frac{3}{2}x\right)^3$

Calculating the binomial coefficient:

$\binom{\frac{2}{3}}{3}=\frac{\frac{2}{3}\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)}{3\times2\times1}$

$=\frac{\frac{2}{3}\cdot\left(-\frac{1}{3}\right)\cdot\left(-\frac{4}{3}\right)}{6}$

Now, simplifying:

$=\frac{2^{\frac{2}{3}}\cdot\left(-\frac{2}{27}\right)}{6}\cdot(-1)^{\frac{-7}{3}}\cdot\left(\frac{27}{8}\right)x^3$

$=-2^{\frac{2}{3}}\cdot\frac{8}{27}\times\frac{27}{8}\times\frac{1}{6}x^3$

Finally, simplifying gives:

$=\frac{-\sqrt[3]{4}}{6}x^3$

So, the 4th term in the expansion is $-\frac{\sqrt[3]{4}}{6}x^3$.

Question33

The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

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Options:

A. 8

B. 9

C. $\frac{80}{9}$

D. $\frac{29}{3}$

Answer: C

Solution:

We have, to find coefficient of x^3 if the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$.

The function is of the type $\left(ax^p + \frac{b}{x^q}\right)^n$, then

$r = \frac{np-k}{p+q} = \frac{5\times3-5}{3+2} = \frac{5(3-1)}{5} = 2$

Coefficient of $T_{r+1} = {}^n c_r a^{n-r} \cdot b^t$

Coefficient of $T_3 = 5C_2 \cdot (2)^{5-2} \cdot \left(\frac{1}{3}\right)^2$

$$= \frac{51}{3!\cdot 2!} \times 2^3 \times \frac{1}{9}$$
$$= \frac{5\times 4}{2} \times 2^3 \times \frac{1}{9} = \frac{80}{9}$$

Question34

Numerically greatest term in the expansion of $(5+3x)^6$ When, $x = 1$, is

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Options:

A. $3^5 \times 5^3$

B. $3^3 \times 5^5$

C. $3^2 \times 5^5$

Answer: B

Solution:

To find the numerically greatest term in the expansion of $(5+3x)^6$ when $x=1$, we start by analyzing the general term in the binomial expansion:

$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot a^r$

For our specific case:

$T_{r+1} = {}^6C_r \cdot 5^{6-r} \cdot (3x)^r$

We are looking for $\frac{T_{r+1}}{T_r}$ to determine when the terms start decreasing:

$$\begin{aligned}\frac{T_{r+1}}{T_r} &= \left(\frac{6-r+1}{r}\right) \cdot \frac{3x}{5} \\ &= \frac{21-3r}{5r} \quad \text{(by substituting } x=1\text{)}\end{aligned}$$

Now, we analyze when $T_{r+1} > T_r$:

$$\begin{aligned}21-3r &> 5r \\ 8r &< 21 \\ r &< 2.625\end{aligned}$$

Thus, to find the numerically largest term, we evaluate r when $T_{r+1} > T_r$ and $T_{r+1} < T_r$. Specifically, when:

$$\begin{aligned}r < 2.625 &: T_{r+1} > T_r \\ r > \frac{21}{8} &: T_{r+1} < T_r\end{aligned}$$

Now let's calculate T_3 and T_4:

For T_3:

$$\begin{aligned}T_3 &= {}^6C_2 \cdot 5^4 \cdot (3)^2 \cdot 1^2 \\ &= 15 \cdot 625 \cdot 9 \\ &= 5^5 \cdot 3^3\end{aligned}$$

For T_4:

$$\begin{aligned}T_4 &= {}^6C_3 \cdot 5^3 \cdot (3)^3 \cdot 1^3 \\ &= 20 \cdot 125 \cdot 27 \\ &= 4 \times 5^3 \times 3^3\end{aligned}$$

The calculations show that the largest term is the 3rd term (T_3), which equals $5^5 \times 3^3$. Thus, the numerically greatest term in the expansion is $5^5 \times 3^3$.

Question35

The square root of independent term in the expansion of $\left(2x^2 + \frac{5}{x}\right)^5$ is

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Options:

A. $\frac{15}{\sqrt{10}}$

B. $\frac{10}{\sqrt{15}}$

C. $\frac{30}{\sqrt{5}}$

D. $\frac{20}{\sqrt{5}}$

Answer: C

Solution:

$$T_{r+1} = {}^{36}C_r\left(\frac{2x^2}{5}\right)^{10-r} \times \left(\sqrt{\frac{5}{x}}\right)^r$$

$${}^{10}C_r\frac{(2)^{32-}}{5^{10-r-\frac{1}{2}}} \times x^{30-3-\frac{r}{2}}$$

$$= {}^{10}C_r\frac{(2)^{10-1}}{5^{31-\frac{y}{2}}} \times x^{30-\frac{y}{2}}$$

Term will be independent if

$20 - \frac{5r}{2} = 0 \Rightarrow r = 8$

Independent term

$$= {}^{10}C_3\frac{(2)^{10-1}}{5}$$

$$= {}^{10}C_2 \times 2^2 \times 5^3 = 4500$$

Square root of $4500 = 30\sqrt{5}$

Question36

The coefficient of x^5 in $\left(3 + x + x^2\right)^6$ is

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Options:

A. 18

B. 540

C. 0

D. 2178

Answer: D

Solution:

We have, $\left(3+x+x^2\right)^6$

General term of the expansion

$=\frac{6!}{r!s!t!}\times 3^r\times x^5\times\left(x^2\right)^t$

Where,

$r+s+t=6$

$=\frac{6!}{r!s!t!}\times 3^r\times x^{5+2t}$

For the coefficient of x^5,

$s+2t=5$

But $r+s+t=6$

$r+5-t=6$

$r-a=1+t$ and $s=5-2t$

Now, $t=0$, then $r=1, s=5$

$t=1,$ then $r=2, s=3$

$t=2$ then $r=3, s=1$

$\therefore$ There are only 3 terms that contain x^5

$\therefore$ Coefficient of x^5

$=\frac{6!}{0!1!5!}\times 3^1+\frac{6!}{1!2!3!}\times 3^2+\frac{6!}{2!3!!!}\times 3^3$

$=18+540+1620=2178$

Question37

The absolute value of the difference of the coefficients of x^4 and x^6 in the expansion of $x^2-2x^2+(x+1)^4(x^2-1)^2$, is

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Options:

A. 13

B. 4

C. 9

D. 1

Answer: A

Solution:

To determine the absolute value of the difference between the coefficients of x^4 and x^6 in the expansion of the given expression, we start with:

$x^2 - 2x^2 + (x+1)^4(x^2-1)^2$

We simplify the function as follows:

$f(x) = \frac{2x^2}{(x^2+1)(x^2+2)}$

Expanding, we have:

$f(x) = 2x^2(x^2+1)^{-1}(x^2+2)^{-1}$

Simplifying further gives:

$f(x) = \frac{2x^2}{2}\left(1+x^2\right)^{-1}\left(1+\frac{x^2}{2}\right)^{-1}$

This can be expanded into:

$f(x) = x^2[1 - x^2 + x^4 - x^6 + \cdots] \cdot \left[1 - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^4}{8} + \cdots\right]$

Now, let's find the coefficients.

For x^4:

Coefficient calculation:

$\left[-\frac{1}{2} - 1\right] = \frac{-3}{2}$

For x^6:

Coefficient calculation:

$\left[\frac{1}{4} + \frac{1}{2} + 1\right] - \frac{7}{4}$

Now calculate the absolute difference:

$\text{Difference} = \left|\frac{7}{4} - \left(\frac{-3}{2}\right)\right| = \left|\frac{7}{4} + \frac{3}{2}\right|$

$= \left|\frac{7}{4} + \frac{6}{4}\right| = \left|\frac{13}{4}\right| = \frac{13}{4}$

This process allows us to correctly determine the absolute value of the difference between the coefficients of x^4 and x^6 in the expansion.

Question38

The least value of n so that ${}^{(n-1)}C_3 + {}^{(n-1)}C_4 > {}^{n}C_3$

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Options:

A. 11

B. 9

C. 8

D. 7

Answer: C

Solution:

Given the inequality $\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$:

We start with the given expression:

$\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$

Using the properties of combinations, we know that:

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Applying this, we get:

$\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$

This simplifies to:

$\frac{(n-1)!}{3!(n-4)!} + \frac{(n-1)!}{4!(n-5)!} > \frac{n!}{3!(n-3)!}$

Rewriting and dividing both sides by common terms:

$\frac{1}{4!} + \frac{1}{(n-4)} > \frac{1}{n-3 \cdot n-4}$

Simplifying further:

$\frac{1}{4} > \frac{1}{(n-3)(n-4)}$

Taking the reciprocal, we have:

$4 < (n-3)(n-4)$

To find the minimum value of n:

$(n-3)(n-4) > 4$

Solving this inequality, we find:

$n - 3 > 4$

Thus:

$n > 7$

The smallest integer n satisfying the inequality is $n = 8$.

Hence, the least value of n is 8.
